A Method for Design of Feedback Control To Limit Missile Dispersion

D.H. Platus*
The Aerospace Corporation, El Segundo, Calif.
and
Julian Wolkovitch†
Vought Corporation, Dallas, Texas

A method for the design of feedback control to limit cross-range dispersion of spinning missiles is described. The missile transverse velocity in the cross plane is selected as the cost function, and optimal linear feedbacks are derived that minimize this velocity in the presence of aerodynamic disturbances that cause lift nonaveraging. A closed-form solution is obtained for the open- and closed-loop missile response to simple disturbance moments. Cross-range dispersion control complements control of range error by angle-of-attack control of drag. The resulting two-loop system can limit major sources of missile dispersion.

Nomenclature

A,B,C,D,E,F	= feedback gains
a,b,c,d,e,f	= transformed feedback gains, Eq. (11)
c_0,c_1,c_2,d_0,d_1,d_2	= Eq. (19)
I	= pitch or yaw moment of inertia; cost function, Eq. (17)
I_x	= roll moment of inertia
ℓ	= characteristic length, I/mx_{st}
L	= lift force
L_θ	= lift force derivative
m	= missile mass
m_p,m_y	= transformed pitch and yaw disturbance moments, Eq. (12)
$m_{\delta_p},m_{\delta_y}$	= transformed pitch and yaw control moments, Eqs. (13) and (14)
M_p,M_y	= pitch and yaw disturbance moments
$M_{\delta_p},M_{\delta_y}$	= pitch and yaw control moments
p	= roll rate
s	= Laplace transform variable
t	= time
v	= Y component of transverse velocity
V	= transverse velocity $v+iw$
ΔV	= net change in average transverse velocity
w	= Z component of transverse velocity
W_1,W_2,W_3	= weighting constants
x_{st}	= static margin (distance of center of pressure aft of center of mass)
Y,Z	= cross-plane coordinates
α	= $\theta_+/\bar{\theta}$
$\delta(t)$	= unit impulse function
θ	= angle of attack (Euler angle)
$\bar{\theta}$	= mean angle of attack
θ_+	= angle-of-attack perturbation
λ	= $\dot{\psi}_+/\omega_n$
λ_I	= Eq. (22)
τ	= nondimensional time $\omega_n t$
ψ	= precession angle (Euler angle)
$\dot{\psi}$	= precession rate
$\dot{\psi}_+$	= precession rate perturbation
ω_n	= undamped natural pitch frequency

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*Senior Staff Scientist, The Ivan A. Getting Laboratories. Associate Fellow AIAA.

†Senior Staff Engineer. Member AIAA.

I. Introduction

DISPERSION of missiles can be separated into two principal categories: 1) cross-range dispersion resulting from lift nonaveraging, and 2) up- or downrange error associated with drag uncertainty and atmospheric variations such as winds. Winds also contribute to cross-range dispersion. Control of up- or downrange error by compensating for drag uncertainty (drag modulation) has been treated previously. It has been shown that very little energy is required to effect large changes in drag by yaw-moment control of angle of attack.[1,2] The missile is controlled in a circular coning motion in which the drag is a strong function of the coning half-angle (angle of attack). Changes in drag caused by extraneous sources, as detected by drag deceleration, can be compensated for by appropriately increasing or decreasing the angle of attack.

Cross-range dispersion resulting from lift nonaveraging is one of the greatest contributors to missile impact error. The sources of such dispersion are quite varied and include launch errors caused by muzzle disturbance,[3] small mass and configurational asymmetries,[4-7] and asymmetric boundary-layer transition (in the case of ballistic re-entry vehicles).[8] It is well known that dispersion caused from variations in body-fixed asymmetries varies approximately inversely with roll rate,[5,8] and that such dispersion can be controlled by maintaining the roll rate at a sufficiently large steady value (roll control). However, certain forms of disturbance moments can adversely affect the lift-vector precession rate and produce lift nonaveraging dispersion somewhat independent of the roll rate.[8]

In this paper, a simple method is described for the design of feedback control to limit cross-range dispersion caused by lift nonaveraging. A simplification to three degrees of freedom is afforded by the assumptions that the lift force derivative is

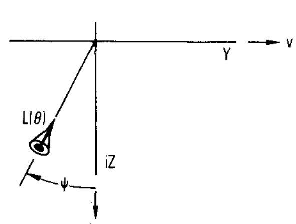

Fig. 1 Transverse velocity.

slowly varying relative to the frequency of disturbances that cause lift nonaveraging and that lateral translational contributions to angle of attack are negligible. Control of lift-nonaveraging dispersion complements drag control to limit up- and downrange errors. With the exception of cross-range dispersion caused by winds, the resulting two-loop system can limit major sources of missile dispersion.

II. Analysis

Control Equations

Cross-range dispersion resulting from lift nonaveraging can be described in terms of the complex missle transverse velocity $V=v+iw$ in a plane normal to the average flight path. This velocity is defined in terms of the total angle of attack θ and lift-vector precession angle ψ, according to[8] (Fig. 1)

$$V(t)=V(0)-\frac{iL_\theta}{m}\int_0^t \theta e^{i\psi}\mathrm{d}t \tag{1}$$

where L_θ is the lift force derivative and m is the missile mass. If the average value of the transverse velocity $V(t)$ is set equal to zero prior to any disturbance in θ and ψ that causes lift nonaveraging, then the cross-range dispersion resulting from a disturbance is proportional to the magnitude of the average value of V at sufficiently large time t after the disturbance.

The behavior of the angles θ and ψ is described approximately by the undamped equations of motion for a slowly rolling (p = const), axisymmetric missile[1]:

$$\ddot{\theta}+(\omega_n^2-\dot{\psi}^2)\theta=M_p/I+M_{\delta_p}/I \tag{2}$$

$$\theta\ddot{\psi}+2\dot{\theta}\dot{\psi}=M_y/I+M_{\delta_y}/I \tag{3}$$

where M_p and M_y are aerodynamic pitch and yaw disturbance moments in the wind-reference axes, and M_{δ_p} and M_{δ_y} are applied control moments.‡ The control problem is to define the moments M_{δ_p} and M_{δ_y} in terms of appropriate state variables that minimize the net transverse velocity V resulting from some disturbance. The problem is that of an optimal regulator with a cost function equal to the magnitude of the integral in Eq. (1).

It is assumed that the missile is initially untrimmed in circular coning motion, which permits a sufficient level of drag control by either increasing or decreasing the angle of attack.[1] Cross-range dispersion is then controlled by minimizing the coupling between angle-of-attack and precession angle perturbations that cause lift nonaveraging in accordance with Eq. (1). The equations of motion, Eqs. (2) and (3), can be linearized in terms of small perturbations about the quasisteady values $\theta=\bar{\theta}$ and $\dot{\psi}=\omega_n$.[1] With $\theta=\bar{\theta}+\theta_+$ and $\dot{\psi}=\omega_n+\dot{\psi}_+$ substituted in Eqs. (2) and (3) and higher-order terms neglected (where θ_+ and $\dot{\psi}_+$ are small perturbations), the resulting linear control equations can be written as

$$\ddot{\theta}_+-2\omega_n\bar{\theta}\dot{\psi}_+=M_p/I+M_{\delta_p}/I \tag{4}$$

$$2\omega_n\dot{\theta}_++\bar{\theta}\ddot{\psi}_+=M_y/I+M_{\delta_y}/I \tag{5}$$

where the control moments are assumed to be linear functions of the state variables θ_+, $\dot{\theta}_+$, and $\dot{\psi}_+$, according to

$$M_{\delta_p}/I=-A\theta_+-B\dot{\theta}_+-C\dot{\psi}_+ \tag{6}$$

‡Aerodynamic damping is omitted for simplicity, and the roll rate is assumed to be small such that $I_x p/2I \ll \omega_n$. Omission of damping does not alter the validity of the proposed method for cross-range dispersion control, and damping can be included readily by modifying the left side of Eqs. (2) and (3) in accord with Eqs. (1) and (2) of Ref. 1, respectively. Damping does influence the yaw moment required to maintain a specified level of angle of attack in circular coning motion for drag control, as explained in Ref. 1.

$$M_{\delta_y}/I=-D\theta_+-E\dot{\theta}_+-F\dot{\psi}_+ \tag{7}$$

It is convenient to nondimensionalize the control equations by defining new variables§

$$\alpha\equiv\theta_+/\bar{\theta},\qquad \lambda\equiv\dot{\psi}_+/\omega_n,\qquad \tau\equiv\omega_n t \tag{8}$$

If these variables are substituted in Eqs. (4-7), since $\mathrm{d}/\mathrm{d}t=\omega_n\mathrm{d}/\mathrm{d}\tau$, and the Laplace transform with respect to τ defined by

$$\mathcal{L}(\ )=\int_0^\infty (\ )e^{-s\tau}\mathrm{d}\tau \tag{9}$$

is taken, then the control equations can be written as

$$\begin{bmatrix} s^2+bs+a & c-2 \\ (2+e)s+d & s+f \end{bmatrix}\begin{bmatrix}\alpha \\ \lambda\end{bmatrix}=\begin{bmatrix}m_p \\ m_y\end{bmatrix} \tag{10}$$

where the nondimensional feedback gains a, b, c, ... are defined by

$$a\equiv A/\omega_n^2,\qquad b\equiv B/\omega_n,\qquad c\equiv C/\omega_n\bar{\theta}$$
$$d\equiv D/\omega_n^2,\qquad e\equiv E/\omega_n,\qquad f\equiv F/\omega_n\bar{\theta} \tag{11}$$

and the nondimensional transformed disturbance moments are

$$m_p\equiv\frac{1}{\omega_n^2\bar{\theta}I}\mathcal{L}(M_p),\quad m_y\equiv\frac{1}{\omega_n^2\bar{\theta}I}\mathcal{L}(M_y) \tag{12}$$

The transformed nondimensional control moments, Eqs. (6) and (7), can similarly be written

$$m_{\delta_p}=-[(a+bs)\alpha(s)+c\lambda(s)] \tag{13}$$

$$m_{\delta_y}=-[(d+es)\alpha(s)+f\lambda(s)] \tag{14}$$

The transverse velocity, Eq. (1), can be written in nondimensional form with the substitutions $\theta=\bar{\theta}(1+\alpha)$ and

$$\psi=\int_0^t(\omega_n+\dot{\psi}_+)\mathrm{d}t=\int_0^\tau(1+\lambda)\mathrm{d}\tau \tag{15}$$

which yields

$$V(\tau)=V(0)-i\ell\omega_n\bar{\theta}\int_0^\tau(1+\alpha)\exp\left[i\int_0^\tau(1+\lambda)\mathrm{d}\tau\right]\mathrm{d}\tau \tag{16}$$

where the relation for pitch frequency $\omega_n^2=L_\theta x_{st}/I$ is used, and $\ell=I/mx_{st}$ is a characteristic length. The control problem defined by Eqs. (10) and (16) is that of a linear optimal regulator. The optimal linear feedback gains a, b, c, ... that will minimize the cost function, Eq. (16), are to be determined. It is necessary to impose the additional constraint that the control moments remain within acceptable limits to preclude the trivial case of very large feedbacks. This is done by redefining the cost function

$$I=W_1VV^*+W_2\int_0^\infty m_{\delta_p}^2\mathrm{d}\tau+W_3\int_0^\infty m_{\delta_y}^2\mathrm{d}\tau \tag{17}$$

where V^* is the complex conjugate of V, m_{δ_p} and m_{δ_y} are the nondimensional control moments defined by Eqs. (13) and (14), and W_1, W_2, and W_3 are suitable weighting functions.

§This step is not essential for the analysis, but it facilitates the numerical evaluation described later.

Solution for Linear Optimal Control

The solution to Eq. (10) for $\alpha(s)$ and $\lambda(s)$ is of the form

$$\alpha(s) = N_1(s)/D(s), \qquad \lambda(s) = N_2(s)/D(s) \tag{18}$$

where $N_1(s)$, $N_2(s)$, and $D(s)$ are polynomials in s of order depending on the form of the disturbance moments m_p and m_y and with coefficients containing the feedback gains $a, b, c, \ldots$ to be determined. The control moment terms in Eq. (17), in the form of integral-square values, can be evaluated readily from the results of Eq. (18) in terms of Phillips integrals.[9,10]

For example, if the system is third order and the pitch-control moment m_{δ_p}, Eq. (13), has the form

$$m_{\delta_p} = \frac{c_2 s^2 + c_1 s + c_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0} \tag{19}$$

then the integral

$$I_3 = \int_0^\infty m_{\delta_p}^2 \mathrm{d}\tau$$

has the value

$$I_3 = \frac{c_2^2 d_0 d_1 + (c_1^2 - 2c_0 c_2) d_0 d_3 + c_0^2 d_2 d_3}{2 d_0 d_3 (d_1 d_2 - d_0 d_3)} \tag{20}$$

Evaluation of the transverse velocity term VV^* is much more difficult because of the nonlinear coupling between α and λ in Eq. (16). A reasonable approximation to Eq. (16) can be obtained, however, by writing it in the form

$$V(\tau) = V(0) - i\ell\omega_n \bar{\theta} \int_0^\tau [1 + \alpha(\tau)] \exp[i\lambda_1(\tau)] e^{i\tau} \mathrm{d}\tau \tag{21}$$

where

$$\lambda_1(\tau) = \int_0^\tau \lambda(\tau)\mathrm{d}\tau, \qquad \lambda_1(s) = \frac{\lambda(s)}{s} \tag{22}$$

and the upper limit of the integral in Eq. (21) is taken to be sufficiently large to include the net change in $V(\tau)$ resulting from perturbations $\alpha(\tau)$ and $\lambda(\tau)$. Without loss of generality, this upper limit can be taken as ∞, and Eq. (21) becomes the Fourier transform of the function $f(\tau)$:

$$V = V(0) - i\ell\omega_n \bar{\theta} \int_0^\infty f(\tau) e^{i\tau} \mathrm{d}\tau \tag{23}$$

where

$$f(\tau) = [1 + \alpha(\tau)] \exp[i\lambda_1(\tau)] \tag{24}$$

Since $\alpha(\tau)$ and $\lambda(\tau)$ are, in general, small perturbations, according to their definitions, Eq. (8), the exponential in Eq. (24) can be expanded to obtain for $f(\tau)$ the approximation

$$f(\tau) \approx 1 + \alpha(\tau) + i\lambda_1(\tau) + i\alpha(\tau)\lambda_1(\tau) - \tfrac{1}{2}[\lambda_1(\tau)]^2 + \cdots \tag{25}$$

Equation (23) can be evaluated in terms of the Laplace transform of $f(\tau)$, Eq. (9), with $s = -i$. The constant term in Eq. (25) is the contribution to $V(t)$ from $\bar{\theta}$ prior to a disturbance that causes $\alpha(\tau)$ and $\lambda(\tau)$. If $V(0)$ in Eq. (23) is defined as the average value of $V(t)$ prior to a disturbance, both the constant and $V(0)$ can be dropped, since only the net change in V from the perturbation is of interest.[8] Equation (23) then can be written as

$$\begin{aligned}\Delta V &= -i\ell\omega_n\bar{\theta}\mathcal{L}\{\alpha(\tau) + i\lambda_1(\tau) + i\alpha(\tau)\lambda_1(\tau) \\ &\quad - \tfrac{1}{2}[\lambda_1(\tau)]^2 + \cdots\}_{s=-i} \\ &= -i\ell\omega_n\bar{\theta}\{\alpha(s) + i\lambda(s)/s + i\mathcal{L}[\alpha(\tau)\lambda_1(\tau)] \\ &\quad - \tfrac{1}{2}\mathcal{L}[\lambda_1(\tau)]^2 + \cdots\}_{s=-i}\end{aligned} \tag{26}$$

where $\alpha(s)$, $\lambda(s)$, and $\lambda_1(s)$ are given by Eqs. (18) and (22). A first-order solution consists simply of the first two terms, which, from Eq. (18), can be written as

$$\Delta V_{1\text{st ord}} = -i\ell\omega_n\bar{\theta}\left[\frac{N_1(-i) - N_2(-i)}{D(-i)}\right] \tag{27}$$

A second-order solution includes, in addition to Eq. (27), the terms involving the Laplace transform of a product. For the case where $\alpha(s)$ and $\lambda(s)$, given by Eqs. (18) and (22), consist of m and n first-order poles, respectively, and no others, the Laplace transforms are given by[11]

$$\mathcal{L}[\alpha(\tau)\lambda_1(\tau)] = \sum_{k=1}^{m} \frac{N_1(s_k)}{D'(s_k)} \lambda_1(s - s_k) \tag{28}$$

$$\mathcal{L}[\lambda_1(\tau)]^2 = \sum_{j=1}^{n} \frac{N_2(s_j)}{D_1'(s_j)} \lambda_1(s - s_j) \tag{29}$$

where s_k are roots of $D(s) = 0$, and s_j are roots of $D_1(s) \equiv sD(s) = 0$. Thus, the approximate solution for V, given by Eqs. (26-29), is obtained directly from the s-plane solution of the control equations in the form of Eqs. (18) and (22). The optimum feedback gains are those values that minimize the cost function, Eq. (17), according to

$$\frac{\partial I}{\partial a} = 0, \quad \frac{\partial I}{\partial b} = 0, \quad \frac{\partial I}{\partial c} = 0, \cdots \tag{30}$$

Response to Impulsive Yaw Moment

The method is illustrated with a simple example that is amenable to an analytical solution. The response to an impulsive yaw moment

$$M_y(t) = M\delta(t) \tag{31}$$

is considered such that $m_y = M/\omega_n\bar{\theta}I$, from Eq. (12), is a constant, and $M_p = 0$.¶ The solution to Eq. (10) for $\alpha(s)$ and $\lambda(s)$ in the form of Eq. (18) is

$$\alpha(s) = \frac{m_y(2-c)}{D(s)} \tag{32}$$

$$\lambda(s) = \frac{m_y(s^2 + bs + a)}{D(s)} \tag{33}$$

where

$$\begin{aligned}D(s) &= s^3 + (b+f)s^2 + [a + bf \\ &\quad + (2+e)(2-c)]s + af + d(2-c)\end{aligned} \tag{34}$$

The Routh stability criterion[10] requires that the feedback gains satisfy the relations

$$\begin{gathered} af + d(2-c) > 0 \\ a + bf + (2+e)(2-c) > 0 \\ b + f > 0 \\ (b+f)[bf + (2+e)(2-c)] + ab - d(2-c) > 0 \end{gathered} \tag{35}$$

These are satisfied with the feedback gains a, $f \neq 0$, and $b = c = d = e = 0$. Optimum values of these two gains will be found that minimize dispersion in accordance with our cost function, Eq. (17). The perturbation solution, Eqs. (32-34), reduces to

¶Since the unit impulse function $\delta(t)$ has units of $1/t$, it must be nondimensionalized by dividing by ω_n prior to taking the Laplace transform defined in Eq. (9), which accounts for the form $M/\omega_n\bar{\theta}I$ rather than $M/\omega_n^2\bar{\theta}I$ for m_y.

$$\alpha(s) = \frac{2m_y}{D(s)}, \quad \lambda(s) = \frac{m_y(s^2+a)}{D(s)}$$

$$D(s) = s^3 + fs^2 + (a+4)s + af \tag{36}$$

and the first-order solution, Eq. (27) is found to be

$$\Delta V_{1\text{st ord}} = \frac{(M\ell/I)(3-a)[3+a-i(a-1)f]}{(a-1)^2f^2+(a+3)^2} \tag{37}$$

This expression vanishes for $a=3$, independent of f. Hence, to the first-order approximation, dispersion caused from an impulsive yaw moment can be eliminated with two simple feedbacks. In a more general case, the feedbacks would be determined to minimize both the dispersion term VV^* and the control moments in our cost function, Eq. (17), with suitable weighting of each term. Since the gain $a=3$ eliminates dispersion, to the first-order approximation, and a nonzero value of the gain f is required for stability, the magnitude of f is determined such as to cause equal integral-square values of the control moments in order to illustrate the method. The control moments, Eqs. (13) and (14), with $a=3$, $b=c=d=e=0$, and the solution, Eq. (36), can be written as

$$m_{\delta_p} = -\frac{2a\, m_y}{D(s)}, \quad m_{\delta_y} = -\frac{fm_y(s^2+a)}{D(s)} \tag{38}$$

where $D(s)$ is defined in Eq. (36). The integral-square values of these moments are given by the I_3 integral, Eq. (20), where the coefficients c_n and d_n are defined for each moment by Eqs. (36) and (38). If the numerators of Eq. (20) are equated for the two moments (since the denominators are the same), the nondimensional gains a and f must satisfy the relation $f=\sqrt{a}$ for equal integral-square values of the control moments. Thus, the optimal linear first-order values of our two feedback gains for the assumed cost function are $a=3$ and $f=\sqrt{3}$. These values should result in zero dispersion, to the first-order approximation, and require equal integral-square values of the control moments. The open-loop dispersion caused by an impulsive yaw moment M is obtained readily from Eq. (26) and the perturbation solution, Eqs. (32-34), with all the feedbacks equal to zero. The first-order approximation to this dispersion is found from Eq. (37) with $a=f=0$ to give the simple result

$$\Delta V_{\text{op lp 1st ord}} = M\ell/I \tag{39}$$

III. Numerical Evaluation

The equations of motion, Eqs. (2) and (3), were integrated numerically to obtain both the open- and closed-loop responses to an impulsive negative yaw moment with the simple optimal feedbacks just derived. The results are compared with first- and second-order analytical approximations to the dispersion velocity. Angle of attack, precession rate, and transverse velocity histories are shown in Figs. 2-7 for the open- and closed-loop responses. The inputs and system parameters are as follows:

$I \quad = 20$ slug-ft^2
$\ell \quad = 6.566$ ft
$\omega_n \quad = 40$ rad/s
$\bar{\theta} \quad = 2$ deg
$a \quad = 3$
$f \quad = \sqrt{3}$
$b \quad = c=d=e=0$
$M \quad = \int M_y dt = -6.092$ ft-lb-s

The feedback gains used in the simulation of Figs. 5-7 ($a=3$, $f=\sqrt{3}$) were determined to give zero net dispersion velocity on the basis of the first-order approximation, Eq. (37). The closed-loop velocity, computed from a numerical integration of the nonlinear equations (2) and (3), is shown in Fig. 7. The feedback control limits the dispersion to approximately 3% of the open-loop value. The second-order contributions to the net dispersion velocity, Eq. (26), can be computed from the relations, Eqs. (28) and (29), for the Laplace transform of a product and from the control solutions, Eq. (36). The second-order terms are found to be

$$\Delta V_{\text{op lp 2nd ord}} = \frac{\ell}{15\,\omega_n \bar{\theta}}\left(\frac{M}{I}\right)^2 = 0.0291 \text{ ft/s} \tag{40}$$

$$\Delta V_{\text{clsd lp 2nd ord}} = \frac{\ell}{\omega_n \bar{\theta}}\left(\frac{M}{I}\right)^2 (0.0509 - 0.0283i)$$

$$= 0.0222 - 0.0123i \text{ ft/s} \tag{41}$$

which should be added to the first-order solutions, Eqs. (39) and (37), of -2.00 ft/s and zero, respectively. The results are summarized in Table 1, which is a comparison of the open- and closed-loop theoretical values with those computed from a numerical integration of the nonlinear equations of motion.

IV. System Implementation

Implementation of a control system to sense the required state variables and to generate wind-referenced pitch and yaw control moments has been discussed in conjunction with drag control systems.[1,2] The principal feature of such systems, which differ from more conventional aerodynamic control

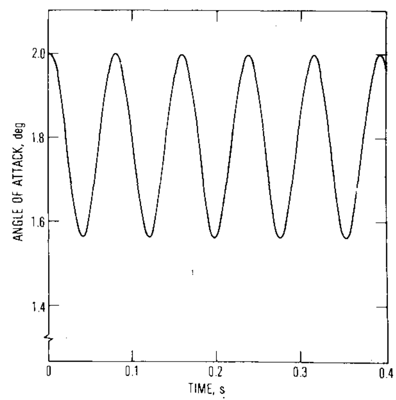

Fig. 2 Open-loop angle-of-attack response to impulse.

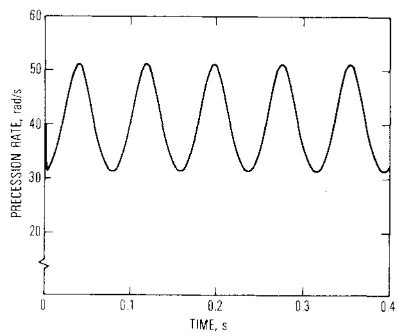

Fig. 3 Open-loop precession rate response to impulse.

Table 1 Net dispersion velocity, ft/s

	Open loop	Closed loop
First-order approximation	2.00	0
First-and-second-order approximations	1.97	0.025
Numerical integration	2.00	0.067

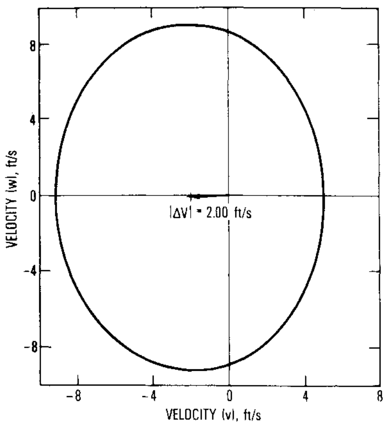

Fig. 4 Open-loop dispersion velocity response to impulse.

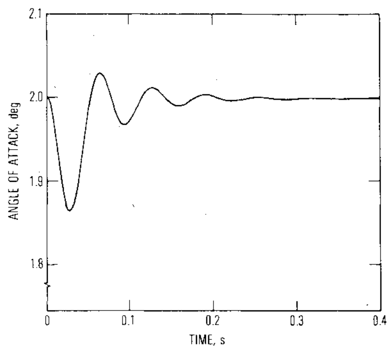

Fig. 5 Closed-loop angle-of-attack response to impulse.

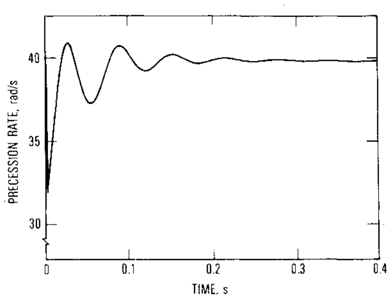

Fig. 6 Closed-loop precession rate response to impulse.

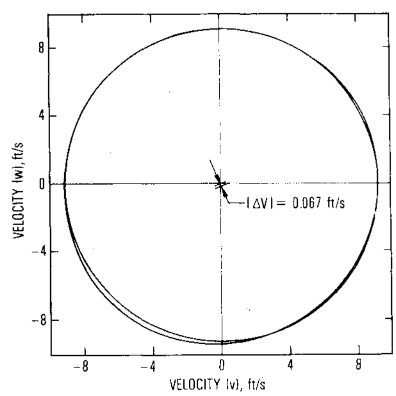

Fig. 7 Closed-loop dispersion velocity response to impulse.

systems, is the requirement to generate aerodynamic control moments in the wind plane, which rotates relative to body-fixed axes at a rate approximately equal to the difference between the roll rate and the natural pitch frequency when the vehicle is untrimmed. Because relatively small control moments are required to correct for aerodynamic disturbances that otherwise would cause lift nonaveraging, the power requirements are small enough that wind-referenced moments can be generated by modulation of body-fixed control surfaces or reaction jets at the windward-meridian rotation frequency. The state variables selected for this analysis consist of the angle of attack, the pitch rate, and the precession rate. Since the transverse dispersion velocity, Eq. (1), is the integral of the lateral acceleration, a more approriate control variable than total angle of attack is lateral acceleration, which is measured directly from a resolution of body-fixed lateral accelerometers. The pitch rate $\dot{\theta}$ can similarly be obtained from a resolution of body-fixed lateral rate gyro measurements. The precession rate is the difference between the roll rate and the windward-meridian rotation frequency. With the missile untrimmed in a circular coning motion at a quasisteady angle of attack $\bar{\theta}$, a body-fixed lateral accelerometer will measure a strong periodic signal at the windward-meridian rotation frequency. The difference between this signal frequency and the roll rate is the precession rate. Hence, all required control parameters can be obtained from conventional strapped-down sensors.

For control of yaw-moment disturbances only, as discussed in the numerical example, the only feedbacks required are angle of attack (lateral acceleration) and precession rate. For a small or known roll rate, control could be achieved with information derived solely from lateral accelerometers.

V. Conclusions

Cross-range dispersion of spinning missiles resulting from lift nonaveraging can be controlled by application of wind-referenced pitch and yaw moments. A linear optimal control is derived that minimizes transverse dispersion velocity selected as the cost function. An approximation to the dispersion velocity permits the optimal feedback gains to be obtained directly from the *s*-plane solution of linearized control equations by means of classical techniques. The method also yields closed-form solutions for the open- and closed-loop system response to simple disturbances. The control loop, defined to minimize cross-range dispersion, complements control of up- and downrange error by angle-of-attack control of drag. The combined two-loop system can limit major sources of missile dispersion.

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